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LETTER TO THE EDITOR

Towards electrodynamical models of particles

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Abstract. By treating the field of a charge in a stationary orbit as a rotating field no radiation term arises. Stationary circular orbits of positronium are considered from this viewpoint, a new interparticle force being obtained. Bohr quantisation of canonical angular momentum leads to two sets of stationary states, one familiar and the other with properties which permit identification with the neutrino. Systems of one or two neutrinos and an electron afford models for charged pions and muons. Other systems are considered, one with properties which might permit identification with the proton.

A problem which has never been satisfactorily resolved is why an electron in a stationary orbit fails to radiate, despite its acceleration. Stationary orbits are selected by the Wilson–Sommerfeld quantisation conditions, which might be introduced in the capacity of boundary conditions for a continuum of field energy. Whatever the status of these conditions, there is no doubt concerning their efficacy for the discovery of new phenomena; one recalls quantisation of fluxoid in superconductors, most clearly revealed by the Bohm–Aharanov effect for the superconducting medium (Jaklevic *et al* 1965), and quantisation of circulation in superfluids (Rayfield and Reif 1964). For the role of the rules in stochastic electrodynamics see Boyer (1978).

Given a rotating charge, or system of charges, there are two fields to consider: (i) one may employ the Lienard–Wiechert solution of Maxwell's equations to calculate the field in the conventional manner; or (ii) one may transform the field with respect to the co-rotating frame to the laboratory reference system. In the case of uniform linear motion it is immaterial which procedure is adopted; the same field results from either. But in the case of rotational motion different results are obtained unless the correct constitutive properties of the medium are assumed in the situation when background matter rotates.

An example of the difference between the fields obtained by procedures (i) and (ii) is provided by the Oppenheimer–Schiff paradox (Schiff 1939, Corum 1977). Here the field is that of a charged rotating concentric sphere capacitor. Externally to the capacitor the electric field always vanishes, but the magnetic field obtained by method (i) does not vanish whereas that obtained by method (ii) does vanish.

I offer the hypothesis that the field of the electron in a stationary orbit is of type (ii), not type (i) as usually assumed. The absence of a radiation term in the field is then assured. But field lines, and indeed light rays, will be curved, implying non-uniform constitutional properties for the medium (Browne 1977). This is tantamount to proposing that the co-rotating frame, not the laboratory frame, has inertial status—a consequence, evidently, of the quantisation conditions.

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Consider, in the light of the hypothesis, stationary orbits of positronium. Let charges e_1 and e_2 follow a common circular orbit of radius r and diameter r_{12}, which is the interparticle spacing at any instant for the rest frame of the centre of mass. Let the velocities be $\boldsymbol{\beta}_1 c = \boldsymbol{\omega} \times \boldsymbol{r}_1$ and $\boldsymbol{\beta}_2 c = \boldsymbol{\omega} \times \boldsymbol{r}_2$, and introduce the notation $\gamma = (1-\beta^2)^{-1/2}$. Either charge has rest mass m.

With respect to the co-rotating frame S′ the field has 4-potential $(0, 0, 0, \varphi')$, where $\varphi' = e_2/r_{12}$ if we consider the field due to e_2 evaluated at the position of e_1. We rotate this field to a reference system R which rotates relative to S′. Following Corum (1977, 1980) we choose R to be the field of rest frames of particles of a rotating fluid, bearing in mind that the time axes of these tetrads will not be parallel and that the system is anholonomic. The 4-potential with respect to R is obtained by a Lorentz transformation for velocity field $\boldsymbol{\beta}_1$; it is $(\boldsymbol{A}, \varphi)$, where

$$\varphi = \gamma_1 e_2/r_{12}, \qquad \boldsymbol{A} = \gamma_1 \boldsymbol{\beta}_1 e_2/r_{12}. \tag{1}$$

In obtaining fields from these potentials the object of anholonomity makes a contribution (Corum 1977). One finds

$$\boldsymbol{E} = \gamma_1 e_2 \hat{\boldsymbol{r}}_{21}/r_{21}^2, \qquad \boldsymbol{B} = \boldsymbol{\beta}_1 \times \boldsymbol{E}, \tag{2}$$

where $\hat{\boldsymbol{r}}_{21}$ is a unit vector directed from e_2 to e_1. Notice that the velocity entering into these fields is $\boldsymbol{\beta}_1$, not $\boldsymbol{\beta}_2$, because the velocity of the field lines at the position of e_1 is not the velocity of the source charge e_2 but that of the field charge e_1. Corum uses the result $\boldsymbol{B} = \boldsymbol{\beta}_1 \times \boldsymbol{E}$ to resolve the Schiff paradox.

The Lorentz force on e_1 due to the fields (2) is

$$\boldsymbol{F} = e_1(\boldsymbol{E} + \boldsymbol{\beta}_1 \times \boldsymbol{B}) = e_1(1-\beta_1^2)\boldsymbol{E} = (1-\beta_1^2)^{1/2} e_1 e_2 \hat{\boldsymbol{r}}_{21}/r_{21}^2. \tag{3}$$

The equation for circular orbit motion under this force is

$$\gamma\beta^2 mc^2/r = \gamma(1-\beta^2)e^2/r_{12}^2. \tag{4}$$

By introducing $u = d/r_{12}$, where $d = e^2/mc^2 = 2.82 \times 10^{-13}$ cm, and noting that $r_{12} = 2r$, one may simplify (4) to

$$2\gamma^2\beta^2 = u. \tag{5}$$

For the canonical angular momentum of the system we assume $2\boldsymbol{r} \times (\gamma\boldsymbol{\beta}mc + e\boldsymbol{A}/c)$, where $\boldsymbol{A}$ is given by (1). Bohr quantisation of this quantity yields

$$\gamma\beta(1-u) = nu/\alpha, \tag{6}$$

where $\alpha = e^2/\hbar c = 1/137$.

Elimination of u between (5) and (6) yields a quadratic equation for $\gamma\beta$ with roots

$$\gamma\beta = -(n/2\alpha)[1 + \alpha^2/n^2 \pm (1 + 2\alpha^2/n^2)^{1/2}]. \tag{7}$$

To highest order in α^2 the roots, and the corresponding values of u, are

$$\gamma'\beta' = \alpha/2n, \qquad u' = \alpha^2/2n^2, \tag{8a}$$

$$\gamma''\beta'' = -n/\alpha, \qquad u'' = 2n^2/2\alpha. \tag{8b}$$

The negative sign of $\gamma''\beta''$ merely means that the interaction contribution to the angular momentum in (6) is dominant, and it can be eliminated by reversal of β''.

For interaction energy under the velocity-dependent force law (3) we adopt the expression

$$V = e_1(\varphi - \boldsymbol{\beta}_1 \cdot \boldsymbol{A}) = -\gamma_1(1-\beta_1^2)e^2/r_{12}, \tag{9}$$

where one uses (1) for the potentials, noting that $e_1e_2 = -e^2$. The justification for this result will be given in the more detailed work (Browne 1981). Then the total energy of the system is W, where

$$W = 2\gamma mc^2 + V = 2(1-\beta^2)^{1/2}mc^2. \tag{10}$$

Hence the equation of motion (4) has been to obtain $V = -2\gamma\beta^2 mc^2$. The result (10) is in agreement with that obtained by several other authors (Schild 1963, Dorling 1970, Andersen and von Baeyer 1971). Substituting from (8), the two sets of stationary states have energies

$$W' = (2 - \alpha^2/4n^2)mc^2, \tag{11a}$$

$$W'' = (2\alpha/n)mc^2. \tag{11b}$$

The solution (8*a*) and (11*a*) describes the 'atom-like' states of positronium already familiar to us. Orbital motion is nonrelativistic ($\gamma\beta \ll 1$), orbital diameters are large compared with d ($u \ll 1$), and energies differ from $2mc^2$ by order $\alpha^2 mc^2$.

The solution (8*b*) and (11*b*) describes new states, which will be termed 'particle-like'. Orbital motion is ultrarelativistic ($\gamma\beta \gg 1$), orbital diameters are small compared with d ($u \gg 1$), and energies now differ from zero by order αmc^2. Motion, of necessity, is ultrarelativistic when interaction energy greatly exceeds rest energy, a feature of the tightly bound states.

There has been no explicit introduction of spin variables or of vacuum polarisation effects. In Dirac's theory both are implicit in the relativistic equations, and I take the attitude that the same should be true for classical electrodynamics (Browne 1970). For this reason the theory of Schild and Schlosser (1968) is suspect.

Positronium in one of the ultrarelativistic particle-like states will be termed a 'positronium unit' or simply 'unit'. Since $W'' = 7{\cdot}46n^{-1}$ keV the spectroscopy of these states is at x-ray wavelengths, but the probability of radiative interactions will be exceedingly small since r''_{12} is so small compared with the wavelength. Compare the probabilities for 2-photon decay from the $n = 1$ atom-like and particle-like states. The wavenumbers for the emitted photons are k' and k'', where $\hbar ck' = 2mc^2$ and $\hbar ck'' = 2\alpha mc^2$, so that $k''/k' = \alpha$. From (8) we find that $r''_{12}/r'_{12} = \alpha^4/4$. For the ratio of transition probabilities one finds $(k''/k')^3(r''_{12}/r'_{12})^2 = \alpha^{11}/16 = 1{\cdot}96 \times 10^{-25}$. In the case of the atom-like state one knows that the probability of 2-photon decay is 8×10^9 s^{-1} (Heitler 1954), and hence that for the particle-like state is $1{\cdot}6 \times 10^{-15}$ s^{-1}. For comparison, the transition probability for the 21 cm line of hydrogen is $2{\cdot}85 \times 10^{-15}$ s^{-1}, so that if positronium units have abundance comparable with neutral hydrogen in the Galaxy detection should be possible. But there may be a complication; if the unit has spin $\frac{1}{2}$, as suggested below, decay may not occur even at this rate.

Although the net charge of a positronium unit is zero, the unit should possess a magnetic moment because spin moment of the electron and positron will be parallel if the spins are antiparallel. But this moment $\boldsymbol{\mu}''$ will not equal two Bohr magnetons. In the co-rotating frame the effective mass of either charge is m^*, where $2m^*c^2 = 2mc^2 - e^2/r''_{12} \simeq -e^2/r''_{12}$. Assuming a gyromagnetic ratio e/m^*c, one estimates

$$\mu'' = (e/m^*c)(n\hbar) \simeq -ner''_{12}/\alpha = -\alpha ed/2n, \tag{12}$$

where r''_{12} is obtained from (8). Another way to justify this result is to consider the interaction angular momentum introduced by assigning to one charge a mangetic moment $\frac{1}{2}\boldsymbol{\mu}''$; on equating this to $\frac{1}{2}n\hbar$ one obtains for $\boldsymbol{\mu}''$ the result (12).

A finite magnetic moment $\boldsymbol{\mu}''$ seems inconsistent with zero spin. Under certain circumstances the Wilson–Sommerfeld quantum numbers equal an integer plus one-half (Beers 1972, Boyer 1978). If the only non-ignorable coordinate is r, the Bohr quantum number c will equal an integer plus one-half. On such grounds one might consider spin $\frac{1}{2}$ for the positronium unit. Half-integral angular momentum of orbital origin implies a state of imaginary parity.

If one postulates spin $\frac{1}{2}$ for the positronium unit, the system has properties which permit its identification with the neutrino. The identification implies a neutrino rest mass of $2\alpha m/n$ and a neutrino magnetic moment $\alpha ed/2n$ (α^2/n Bohr magnetons). Both these quantities tend to zero as n tends to infinity; the model therefore predicts variable mass and moment for the particle. Since the direction of $\boldsymbol{\mu}''$ relative to the direction of orbital angular momentum has not been specified, the model leaves room for two types of neutrino, presumably with parities +i and −i. Might these be the ν_e and ν_μ? The possible intrinsic parities for Dirac fermions are ±1 and ±i, the parity of a fermion–antifermion pair always being −1 (Roman 1964). Then the antineutrino would have the same parity as the neutrino.

Now, let two positronium units orbit (at diametrically opposed instantaneous positions) around an electron or positron, which remains stationary at the centre of mass, the coupling being spin–orbit in character. Motion of the magnetic moment $\boldsymbol{\mu}''$ (a Lorentz invariant for transverse polarisation) with velocity $\boldsymbol{\beta}c$ gives rise to an electric moment $\boldsymbol{\epsilon}'' = \boldsymbol{\beta} \times \boldsymbol{\mu}''$. In the fields (2) an orbiting unit has potential energy V and experiences force $\boldsymbol{F}$, where

$$V = \boldsymbol{\epsilon}'' \cdot \boldsymbol{E} + \boldsymbol{\mu}'' \cdot \boldsymbol{B} = 0, \qquad |\boldsymbol{F}| = \epsilon''\delta E/\delta r + \mu''\delta B/\delta r = \gamma\beta\mu''e/r^3. \tag{13}$$

In evaluating the gradients of the fields, $\beta(r)$ and $\gamma(r)$ are treated as fields; evidently γ^{-1} changes the constitutional properties of the medium as would a classical permittivity and (equal) permeability.

Under the force (13) the equation for a circular orbit is

$$\gamma\beta m''c^2/r = \gamma\beta\mu''e/r^3. \tag{14}$$

Bohr quantisation of canonical angular momentum now yields

$$2|\boldsymbol{r} \times (\gamma\boldsymbol{\beta}m''c + e\boldsymbol{A}/c)| = \bar{n}\hbar, \tag{15}$$

where $\boldsymbol{A} = \gamma\boldsymbol{\mu}'' \times \boldsymbol{r}/r^3$. On substituting for $\boldsymbol{A}$ and then using (14) to eliminate μ'', one obtains from (15)

$$4\gamma\beta m''cr = \bar{n}\hbar. \tag{16}$$

The ultrarelativistic solution of (14) and (16) is

$$r = d/2, \qquad \gamma = n\bar{n}/4\alpha^2. \tag{17}$$

For the energies of these states one has

$$W = 2\gamma m''c^2 + mc^2 + V = (\bar{n}/\alpha + 1)mc^2. \tag{18}$$

Replacing $\bar{n}$ by $\bar{n}' + \frac{1}{2}$, since the spin of the central charge should be included in (15), one finds $W = (\bar{n}' + \frac{1}{2})mc^2/\alpha + mc^2$. For $\bar{n}' = 1$ this gives $W = 206{\cdot}5mc^2$.

The rest energy of the muon is $206.8mc^2$. Thus the system ('magnetium') has the correct rest energy, the correct spin, and the correct charge for identification with the muon. Moreover simple decomposition into constituents will account for the decay of the muon. Since the spins of the orbiting positronium units must be opposed to satisfy the Pauli principle, and their magnetic moments must be parallel for attraction to the central charge, it follows that one of the orbiting units must be a neutrino of electron type and the other a neutrino of muon type. Hence the decay will be $\mu^+ \to e^+, \bar{\nu}_\mu, \nu_e$ as observed.

Now suppose that only one unit is in orbit around the charge. Despite the unequal masses of the unit and the charge, the centre of mass tends to their midpoint in the ultrarelativistic extreme. Following the same steps as above, one is led to ultrarelativistic state energies $W = 2\bar{n}mc^2/\alpha$. The state $\bar{n} = 1$ has energy $274mc^2$.

The rest energy of the charged pion is $273{\cdot}1mc^2$. Thus, in regard to rest mass, spin and charge, magnetium with one unit has the properties of the charged pion. Decay evidently involves creation of a ν_μ, $\bar{\nu}_\mu$ pair, of which $\bar{\nu}_\mu$ is retained and ν_μ released. This implies that $\pi^+ \to \mu^+, \nu_\mu$, which is the observed decay.

The very different reactivities of the muon and pion can also be understood. It is a simple case of a closed shell. The relationship of the muon to the pion might be compared with that of an inert gas atom to a monovalent metal atom.

Regarding the π^0 one must contemplate a molecular-type complex of two magnetium particles, each with one unit but with opposite charges—that is a complex of π^+ and π_-, the additional rest energy of the second pion being cancelled approximately by the binding energy.

Consider, now, a system ('trionium') in which two electrons orbit (at diametrically opposed instantaneous positions) about a positron or vice versa. Again assuming rotating fields of type (2), it turns out (by following the steps that led to (8) and (11)) that the ultrarelativistic states now have energies $W = 2(1-\beta^2)^{1/2}mc^2 + mc^2 \simeq mc^2$. That is, the mass of the system differs little from that of the central charge. The orbiting charges have ultrarelativistic motion $\gamma''\beta'' = n/\alpha$ and the orbital diameters are $r''_{12} = 3\alpha^2 d/2n^2$.

Now trionium has an orbital current. Coupling to the spin magnetic moment of the central charge is already implicit in the above theory. But suppose that a second pair of orbiting charges are introduced, of sign opposite to the first pair. Assume a larger orbit co-planar with the first one. The second pair couple not only to the net charge of trionium ($\pm e$), but also magnetically to the orbital current. If the orbital current of trionium is treated as a magnetic dipole, no dramatic change of energy occurs, because again one obtains an expression (10). In second order, however, there arises a contribution $5.2nmc^2$ to the energy. Now continue to add pairs of identical charges in further co-planar orbits of increasing radius. It can be shown that the limiting radius tends to d, but calculation of the energy is not simple. One wonders if such a system might account for the proton. It should be extremely stable because charges in each orbit cannot annihilate each other.

For many years I have been of the opinion that the ultimate constituents of matter cannot be other than electrons and positrons, the strong forces of particle physics being basically electromagnetic forces (Browne 1962, 1966). Sternglass (1961, 1965) also seems to have shared this philosophy. However, the problems confronting such a viewpoint are substantial, and my impression now (following a concerted attack over some three years) is that they can be overcome. I take the view that the initial approach must be based on classical dynamics, as was so for atomic physics. Until dynamical systems have been defined classically, the application of quantum electrodynamics is not particularly meaningful. In any case, the quantum electrodynamics of strong interactions ($e^2/r_{12} \gg mc^2$) has not been developed to any extent, owing to the mixing of the positive and negative energy states and the failure of perturbation theory methods.

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